5-4 Videos Guide

5-4a

Theorem (statement and partial proof):

- Green's Theorem (essentially the Fundamental Theorem of Calculus for double integrals): For a region D in $\mathbb{R}^2$ bounded by a positively oriented, simple, closed curve C and a vector field $\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$, if P and Q have continuous partial derivatives on an open region containing D, then
  $\int_C \mathbf{F} \cdot d\mathbf{r} = \oint_C P\,dx + Q\,dy = \int_{\partial D} P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

5-4b

- Notation: $C = \partial D$ is the simple, positively oriented boundary curve of D. The symbol $\oint_C$ is used to indicate positive orientation.

Exercises:

- Evaluate the line integral by two methods: (a) directly and (b) using Green's Theorem.
  - $\oint_C y\,dx - x\,dy,\;$ C is the circle with center the origin and radius 4

5-4c

  - $\oint_C x^2y^2\,dx + xy\,dy,\;$ C consists of the arc of the parabola $y = x^2$ from $(0,0)$ to (1, 1) and the line segments from $(1,1)$ to $(0,1)$ and from $(0,1)$ to $(0,0)$

5-4d

- Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. (Check the orientation of the curve before applying the theorem.)
  $\mathbf{F}(x,y) = \langle e^{-x} + y^2,\, e^{-y} + x^2 \rangle,\;$ C consists of the arc of the curve $y = \cos x$ from $(-\pi/2, 0)$ to $(\pi/2, 0)$ and the line segment from $(\pi/2, 0)$ to $(-\pi/2, 0)$

- Green's Theorem and regions with holes